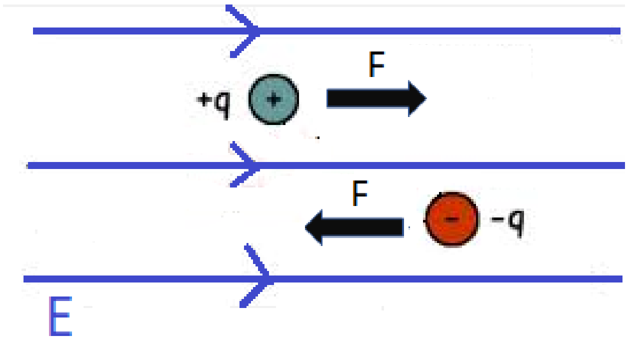
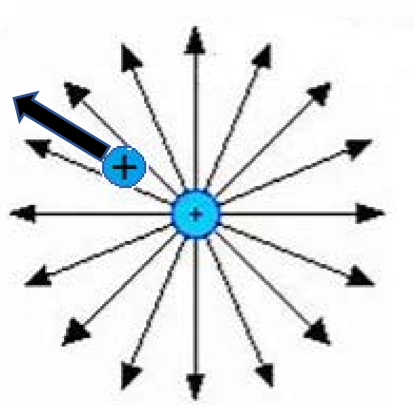
A.4 Electric Force

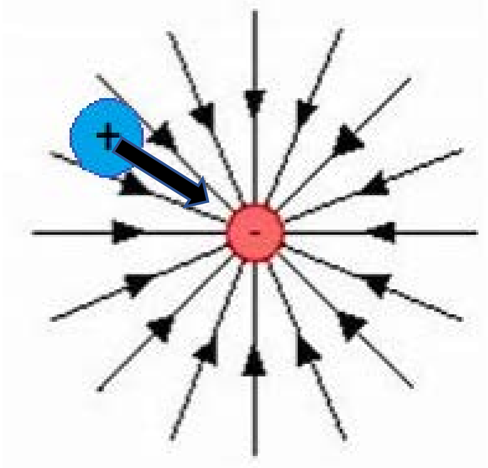
Electric charges interact with one another (repel, attract) via the electric fields they generate. And though we've delineated the different ways to calculate their electric field, we haven't stated how this field interacts with other charges, i.e., what *force* the electric field exerts on the other charges. That all ends today! dun dun dun …….



$$\mathbf{F} = |q|E \begin{cases} \text{in the direction of the } \mathbf{E} \text{ field if q is positive} \\ \text{opposite the direction of the } \mathbf{E} \text{ field if q is negative} \end{cases}$$
$$= q\mathbf{E}$$

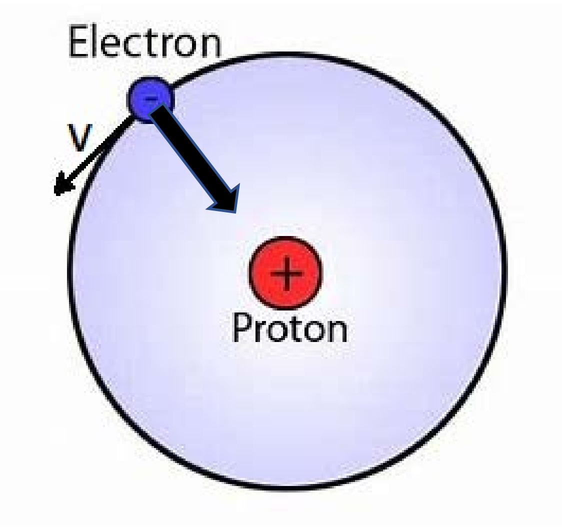
This is 'why' same sign charges repel each other, and opposite sign charges attract.





A.4 Electric Force

Let's revisit our friend the Hydrogen atom. In the ground state, in the classical picture, an electron orbits the proton at distance roughly equal to the Bohr radius *a* = 53pm. And its mass is 9.11×10^{-31}kg.



(a) What force does the proton exert on the electron at that radius?

$$F = |q|E$$
$$= |q_{electron}| \cdot \frac{k|q_{proton}|}{r^2}$$
$$= e \cdot \frac{ke}{r^2}$$
$$= (1.6\times10^{-19}) \cdot \frac{(9\times10^{9})(1.6\times10^{-19})}{(53\times10^{-12})^2} = 8.2\times10^{-8}\,\mathrm{N}$$

(b) Assuming its orbit is circular, what is its velocity as a fraction of light speed?

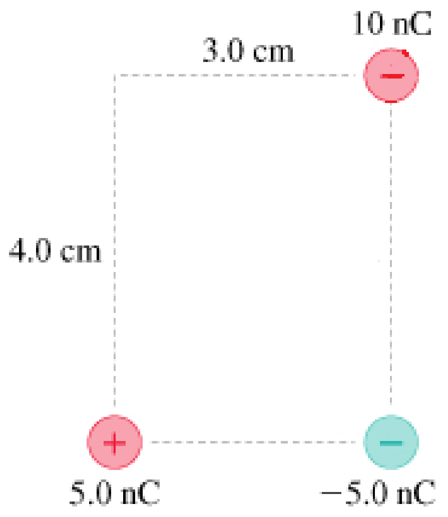
$$\sum F_c = ma_c \qquad a_c = \frac{v^2}{r}$$

$$8.2\times10^{-8}\,\mathrm{N} = (9.11\times10^{-31}\,\mathrm{kg})\left(\frac{v^2}{53\times10^{-12}\,\mathrm{m}}\right)$$

$$v = \sqrt{\frac{8.2\times10^{-8}\,\mathrm{N}}{(9.11\times10^{-31}\,\mathrm{kg})} \cdot (53\times10^{-12}\,\mathrm{m})} = 2.2\times10^{6}\,\mathrm{m/s}$$

$$\frac{v}{c} = \frac{2.2\times10^{6}\,\mathrm{m/s}}{3\times10^{8}\,\mathrm{m/s}} = 0.7\%$$

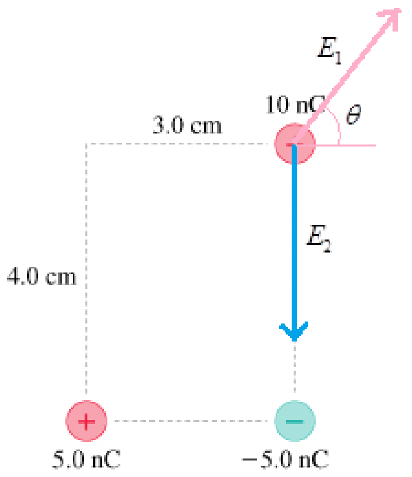
A.4 Electric Force



Working at IBM one Friday afternoon around 4:00, you're so bored you begin wandering around the corridors. Eventually you stumble into the STM room and start making random arrangements of microscopic charges. Your masterpiece is shown over here to the left. Now you want to calculate the force on the 10nC charge.

First we need to get the electric field acting on our 10nC charge….

$$\mathbf{E}_1 = \frac{k|q_1|}{r_1^2}(direction) = \frac{(9\times10^9)(5\times10^{-9})}{(0.03)^2+(0.04)^2}(\cos\theta\hat{\mathbf{i}}+\sin\theta\hat{\mathbf{j}}) = \frac{45}{0.05^2}\left(\frac{0.03}{0.05}\hat{\mathbf{i}}+\frac{0.04}{0.05}\hat{\mathbf{j}}\right) = 1.08\times10^4\hat{\mathbf{i}}+1.44\times10^4\hat{\mathbf{j}}$$



$$\mathbf{E}_2 = \frac{k|q_2|}{r_2^2}(direction) = \frac{(9\times10^9)(5\times10^{-9})}{(0.04)^2}(-\hat{\mathbf{j}}) = -2.8\times10^4\hat{\mathbf{j}}$$

So the field acting on our 10nC charge is

$$\mathbf{E} = \mathbf{E}_1+\mathbf{E}_2 = (1.08\times10^4\hat{\mathbf{i}}+1.44\times10^4\hat{\mathbf{j}})+(-2.8\times10^4\hat{\mathbf{j}}) = 1.08\times10^4\hat{\mathbf{i}}-1.36\times10^4\hat{\mathbf{j}}$$

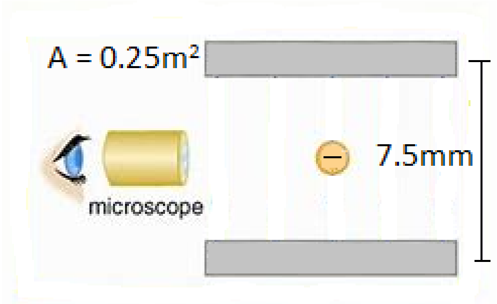
$$= \sqrt{(1.08)^2+(1.36)^2}\times10^4\frac{\text{N}}{\text{C}} \ @\ \tan^{-1}\left(\frac{-1.36}{1.08}\right)^{\circ} = 1.74\times10^4\frac{\text{N}}{\text{C}} \ @\ 51^{\circ}\text{ below horizontal}$$

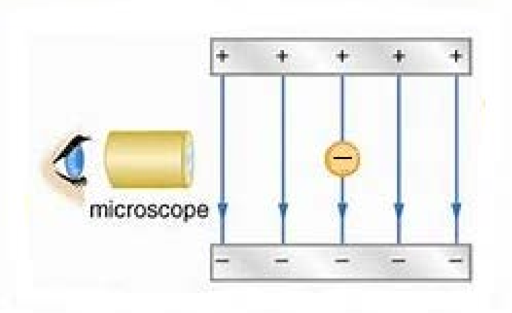
And then finally the force on our 10nC charge is:

$$\mathbf{F} = |q|E \ @\ \text{direction of }\mathbf{E} = (10\times10^{-9})(1.74\times10^4)\ @\ 51^{\circ}\text{ below horiztonal} = 174\,\mu\text{N}\ @\ 51^{\circ}\text{ below horizontal}$$

A.4 Electric Force

A 75μm diameter droplet of oil, having a charge of -2e, is suspended in air directly between two horizontal plates of a parallel plate capacitor. The upward electric force on the droplet is exactly balanced by the downward force of gravity. The oil has a density of 860 kg/m^3, and the capacitor plates are 7.5mm apart. Let's also say the plates each have an area A = 0.25m^2.





(a) which is the positive plate and which the negative plate?

to counter gravity, force must go up, so electric field must go down, so top plate is positive, and bottom negative.

(b) What electric field must exist between the plates?

We can use N2L

$$\sum F = 0$$

$$|q|E - mg = 0$$

$$E = \frac{mg}{|q|}$$

$$E = \frac{(\rho V)g}{2e} = \frac{(860)\cdot\frac{4}{3}\pi\left(\frac{75\times10^{-6}}{2}\right)^3(9.8)}{2(1.6\times10^{-19})} = \frac{(1.9\times10^{-10}\text{ kg})(9.8\,\text{m/s}^2)}{3.2\times10^{-19}\text{C}} = 5.8\times10^9\,\frac{\text{N}}{\text{C}}$$

A.4 Electric Force

(b) how much charge must be pumped onto each of those plates (via battery) to create the electric field sufficient to suspend the oil drop?

$$E = \frac{|\sigma_{top}|}{2\varepsilon_0} + \frac{|\sigma_{bottom}|}{2\varepsilon_0} = \frac{|\sigma|}{2\varepsilon_0} + \frac{|\sigma|}{2\varepsilon_0} = \frac{|\sigma|}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0}$$

$$Q = \varepsilon_0 EA = (8.85\times10^{-12})(5.8\times10^{9})(0.25) = 0.013\text{C} = 13\text{mC}$$

(c) What potential difference between the plates would suffice to create this field (by depositing that charge)

$$\Delta V = -\int_{bottom}^{top} \mathbf{E}\cdot d\mathbf{r} = -\int_0^{7.5\times10^{-3}} 5.8\times10^{9}(-\hat{\mathbf{j}})\cdot dy\hat{\mathbf{j}} = 5.8\times10^{9}\int_0^{0.0075} dy = 4.35\times10^{7}\,\text{V} = 43.5\,\text{MV}$$

(d) If we increased the potential difference to 45MV, how long would it take for the oil droplet to hit the top plate?

Gotta figure out the new field strength… $E = \left|\frac{dV}{ds}\right| = \frac{45\times10^{6}\,\text{V}}{0.0075\text{m}} = 6\times10^{9}\,\frac{\text{V}}{\text{m}}$

Then the acceleration of the charge will be: $a = \frac{\sum F}{m} = \frac{(2e)E - mg}{m} = \frac{(2\cdot1.6\times10^{-19})(6\times10^{9}) - (1.9\times10^{-10})(9.8)}{1.9\times10^{-10}} = 0.31\text{m/s}^2$

And so time will be: $\Delta y = v_0 t + \frac{1}{2}a_y t^2 \longrightarrow \frac{0.0075}{2} = (0)t + \frac{1}{2}(0.31)t^2 \longrightarrow t = \sqrt{\frac{0.0075}{0.31}} = 0.16\text{s}$